

Final Report of Michigan Math and Science Scholars, Graph Theory

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Throughout the MMSS program, I was challenged academically in ways I had never experienced before. Before attending, I expected the program to be academically demanding because I understood that it was designed for students who already excel in mathematical problem solving and share a strong interest in abstract mathematical thinking. However, I quickly realized that it was even more rigorous than I had anticipated. Instead of focusing on finding the correct answer, the course emphasized understanding why an answer is correct and communicating that reasoning through complete, logically sound proofs. This approach completely changed how I viewed mathematics. I discovered that solving a problem is only the first step. Being able to explain every part of the solution clearly and justify every claim is just as important.

The course I attended focused on Graph Theory, a branch of mathematics that models networks and relationships using vertices and edges. Because this was a completely new area of mathematics for me, I learned to represent complex problems through diagrams and analyze the structures within them. We explored topics such as Eulerian and Hamiltonian graphs, trees, combinatorial game theory, graph burning, and the Cops and Robbers game. Although the topics themselves were fascinating, what made the course truly unique was that every idea was developed through proofs and logical reasoning. Instead of memorizing formulas or procedures, we were expected to understand why mathematical statements were true and defend our arguments step by step.

One of the biggest academic challenges was adapting to a style of mathematics that required deep understanding and careful reasoning. Although we did not receive traditional homework every day, I spent many hours in the dorm studying with my roommate, reviewing my notes, thinking through proofs, and working on problems until I truly understood them. Many of the homework questions required creativity and persistence rather than simply applying a method that had been demonstrated in class. There were many occasions when I spent a long time thinking about a single problem before finally discovering an approach that worked. Even after finding a solution, I often had to revise my proof to make sure every step was logically justified.

The program also taught me that making mistakes is an essential part of learning mathematics. Throughout the course, Professor Shaw and the teaching assistants carefully examined our proofs and frequently pointed out gaps in our reasoning or assumptions that were not fully justified. At first, it was frustrating to discover that a proof I believed was complete still contained mistakes. However, I gradually realized that this feedback was one of the most valuable parts of the program. Instead of simply correcting my work, they explained why something was incorrect and showed us common mistakes that mathematicians try to avoid.

Because of this, I became much more careful when writing proofs and learned how to evaluate my own work more critically before considering it complete.

Another valuable aspect of the program was working in small groups. Every afternoon, after Professor Shaw introduced new material during the morning lecture, we worked together in groups to solve challenging proof-based problems. I found these sessions extremely valuable because every student approached problems differently. Sometimes another student's idea would help me see a solution I had completely overlooked, while at other times I was able to explain my own reasoning and help my group understand a proof. Throughout the program, our groups changed several times, allowing me to work with many different people. Before attending MMSS, working closely with others was something I lacked confidence in. In the beginning, I often struggled to express my reasoning clearly, even when I understood the mathematics. By the end of MMSS, I was able to solve many of the challenging problems presented during class and explain my reasoning with much greater confidence. I had also become much more comfortable sharing my ideas, discussing different approaches, and collaborating to solve difficult problems. I believe this is one of the areas in which I grew the most. While I still have much to learn, seeing my own improvement over the six weeks was very rewarding.

Outside of class, I also enjoyed getting to know the other students. One of my favorite places was the student lounge, where many of us gathered after class to relax, play chess, and continue discussing mathematics. Even during lunch, it was common for us to discuss homework problems that none of us had solved yet. These conversations often gave me new perspectives and helped me understand difficult concepts more deeply. I made many new friends throughout the program, and what I appreciated most was being surrounded by people who genuinely enjoyed mathematics. At my home school, it is rare to find classmates who are excited to discuss mathematical ideas outside of class. At MMSS, those conversations happened naturally every day, creating an environment where learning continued even after the lectures ended.

Although I had previous experience living away from home, the residential setting still taught me valuable lessons in independence and time management. The biggest adjustment was managing my time more carefully because there were scheduled check-in times and a structured daily routine. Learning to balance classes, studying, homework, meals, and free time required more self-discipline than I was used to during the school year. At the same time, living with other students made the experience much more enjoyable because we could continue learning from one another outside the classroom. The residential environment made it easy to build friendships and created a strong sense of community that would have been difficult to achieve in a non-residential program.

Professor Shaw played a major role in making this program such a positive experience. His lectures challenged us to think deeply instead of memorizing facts, and he always encouraged us to ask questions whenever something was unclear. He also reminded us that we were always welcome to contact him if we had questions about graph theory or mathematics in

the future. Knowing that someone with so much experience is still willing to support his students long after the course has ended demonstrates how dedicated he is as both a mathematician and an educator. The teaching assistants were equally supportive, constantly providing thoughtful feedback and encouraging us to improve our proofs instead of simply giving us answers.

One of the most valuable lessons I learned came during our final discussions about future education and careers. Professor Shaw spoke about graduate school, research, and the many different opportunities available to students who continue studying mathematics. Although the program helped me realize that becoming a professional mathematician is probably not the career path I want to pursue, I consider that a positive outcome. The experience gave me a much greater appreciation for mathematics and showed me the level of logical thinking and creativity involved in mathematical research. It also helped me understand that the skills I developed during MMSS—critical thinking, logical reasoning, persistence, and communication—are valuable in many different fields, not just mathematics.

Looking back, I believe I grew both academically and personally. Academically, I learned advanced graph theory that I never would have encountered in a traditional high school classroom. More importantly, I learned how mathematicians think by constructing careful proofs and analyzing every logical step. Personally, I became more confident working with others, more comfortable sharing my ideas, and more willing to accept constructive criticism as an opportunity to improve rather than as a sign of failure. These are lessons that will continue to benefit me long after the program has ended.

I would strongly recommend MMSS to future Garwin Family Foundation scholars who enjoy mathematics and are looking for an academic challenge. This program is not focused on memorizing formulas or completing routine calculations. Instead, it encourages students to think deeply, ask questions, collaborate with others, and develop strong proof-writing and problem-solving skills. While the workload can be demanding, the experience is incredibly rewarding because of the knowledge gained, the friendships formed, and the confidence developed throughout the program. Students who are curious, willing to work hard, and excited to explore mathematics beyond the standard high school curriculum will gain a tremendous amount from this experience.

Overall, MMSS has been one of the most rewarding academic experiences I have ever had. It challenged me more than any previous mathematics program while also helping me grow as both a student and a person. The friendships I formed, the guidance I received from Professor Shaw and the teaching assistants, and the mathematical thinking I developed are memories that I will continue to cherish long after the program has ended. Although I may not pursue mathematics as my career, the logical reasoning, communication, collaboration, and perseverance I developed during MMSS will remain valuable throughout my education and whatever profession I choose in the future. I hope to use these skills to solve challenging problems, work effectively with others, and make meaningful contributions in whatever field I

pursue. I am extremely grateful to the Garwin Family Foundation for providing me with the opportunity to attend this program, and I will always appreciate everything I learned during this unforgettable experience.